CASIO

CASIO TEACHING MATERIALS

REAL-LIFE PROBLEMS

with fx-991CW

DIGEST EDITION

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CASIO Teaching Materials

Real-Life Problems with fx-991CW

Introduction

These teaching materials were created with the hope of conveying to many teachers and students the appeal of scientific calculators.

(1) Change awareness (emphasizing the thinking process) and boost efficiency in learning mathematics

- By reducing the time spent on manual calculations, we can have learning with a focus on the thinking process that is more efficient.
- This reduces the aversion to mathematics caused by complicated calculations, and allows students to experience the joy of thinking, which is the essence of mathematics.

(2) Diversification of learning materials and problem-solving methods

- Making it possible to do difficult calculations manually allows for diversity in learning materials and problem-solving methods.

(3) Promoting understanding of mathematical concepts

- By using the various functions of the scientific calculator in creative ways, students are able to deepen their understanding of mathematical concepts through calculations and discussions from different perspectives than before.
- This allows for exploratory learning through easy trial and error of questions.
- Listing and graphing of numerical values by means of tables allows students to discover laws and to understand visually.

Features of these teaching materials

- Makes classes more interesting by using scientific calculators
- Includes a variety of real-life problems in each unit
- Allows a deeper understanding of mathematics
- Enables students to utilize the scientific calculator's functions more skillfully
- Three degrees of difficulty settings (levels 1 to 3)

Better Mathematics Learning with Scientific Calculator

Installation angle of solar panels **Level 1**

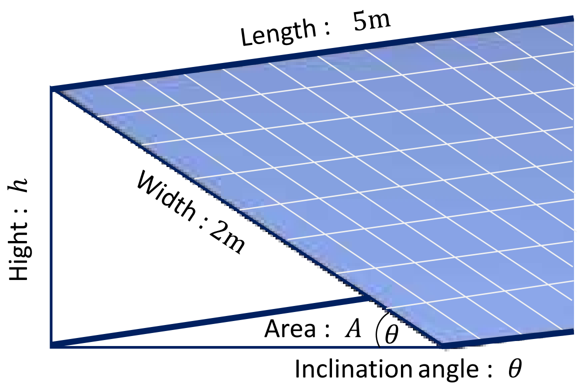


Fig. 1

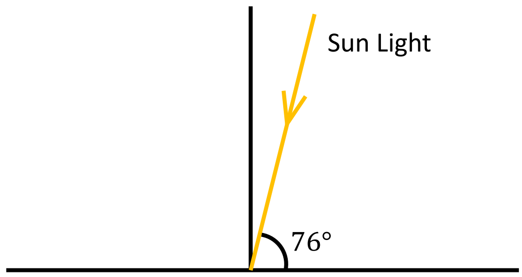


Fig. 2

(1) When installing solar panels as shown in Fig. 1, the power generation efficiency is highest when the sunlight is perpendicular to the panel. When the angle of incidence of the sun is $76°$ as shown in Fig. 2, find the angle θ at which the panel is perpendicular to the sunlight.

$\theta = 90 - 76 = 14°$

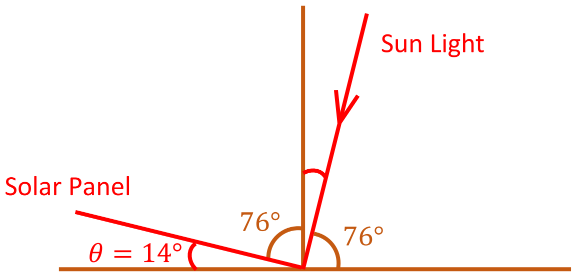
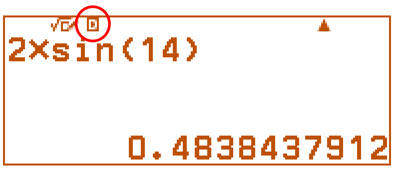


Fig. 3 (For explanation)

(2) In (1), what should the height h [m] of the support column be?
(The width of the solar panels is 2m as shown in Fig. 1.)

$h = 2 \times \sin 14° \approx 0.48\text{m}$



(3) In (1), what is the required footprint of the solar panels?
(The length of the solar panels is 5m as shown in Fig. 1.)

$A = 5 \times 2\cos 14° \approx 9.7\text{m}^2$

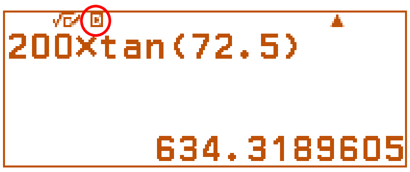
Tower and mountain height surveying **Level 2**

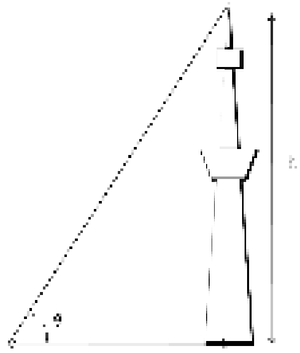
(1) Tower height

At a point 200 [m] from the tower, the elevation angle θ was $72.5°$.

Find the height h [m] of the tower.

$h = 200 \times \tan 72.5 \approx 634$ [m]

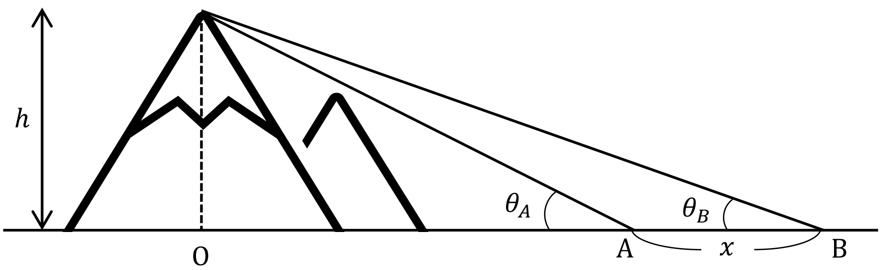




200 [m]

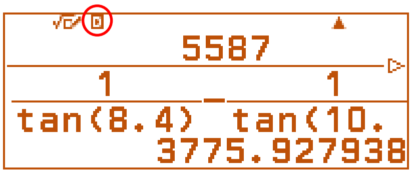
(2) Mountain Height

Unlike with the tower, it is not possible to measure the horizontal distance from the mountain, so the elevation angles were measured at two points A and B. The elevation angle at point A was $\theta_A = 10.7°$ and at point B was $\theta_B = 8.4°$. The distance between A and B was $x = 5{,}587$m. Find the mountain height h [m].



$$x = \mathrm{OB} - \mathrm{OA} = \frac{h}{\tan\theta_B} - \frac{h}{\tan\theta_A} = h\left(\frac{1}{\tan\theta_B} - \frac{1}{\tan\theta_A}\right)$$

$$h = \frac{x}{\frac{1}{\tan\theta_B} - \frac{1}{\tan\theta_A}} = \frac{5{,}587}{\frac{1}{\tan 8.4°} - \frac{1}{\tan 10.7°}} \approx 3{,}776 \text{ [m]}$$



In (2), we may introduce another solution that uses the sine theorem.

How many folds to reach the Moon? Level 1

Given that the distance from the Earth to the Moon is $384{,}400$ km, and that the thickness of the piece of paper is 0.1 mm, find the number of folds required for the thickness of the paper cover the distance to the Moon.

Thickness after x folds: 0.1×2^x mm

0.1×2 mm $= 0.2$ mm (one fold)

0.1×2^2 mm $= 0.4$ mm (two folds)

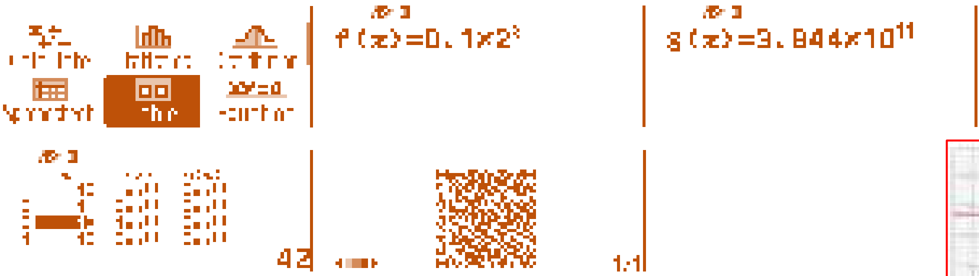
⋮

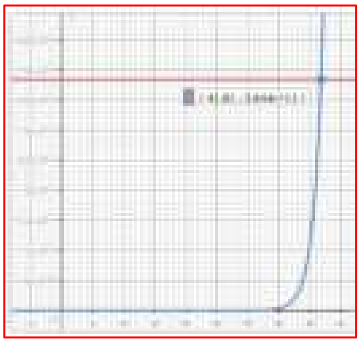
0.1×2^{41} mm $\approx 2.2 \times 10^{11}$ mm (41 folds)

0.1×2^{42} mm $\approx 4.4 \times 10^{11}$ mm (42 folds)

・Distance between the Earth and the Moon: $384{,}400$ km $= 384{,}400 \times 10^6$ mm $= 3.844 \times 10^{11}$ mm

Under [Table], input $f(x) = 0.1 \times 2^x$, and $g(x) = 3.844 \times 10^{11}$. Find the value of x for which $f(x)$ exceeds 3.844×10^{11} mm.



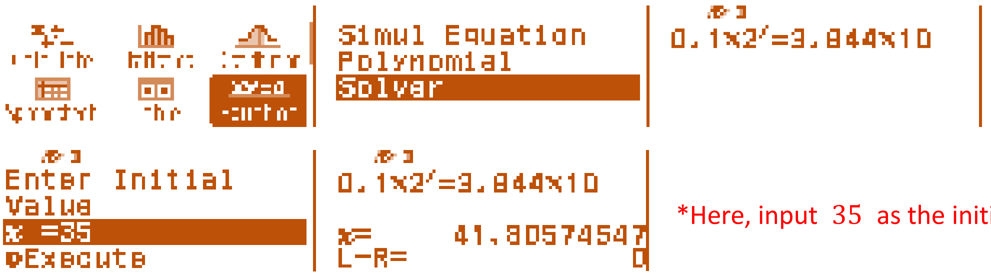


∴ The paper will reach the Moon after 42 folds.

Alternate Solution 1

$0.1 \times 2^x \geq 3.844 \times 10^{11}$

$x \geq 41.8\cdots$



*Here, input 35 as the initial value.

Alternate Solution 2

Rewrite the inequality from Alternate Solution 1 and solve using logarithms.

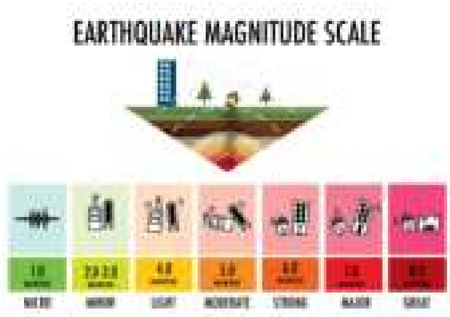
$0.1 \times 2^x \geq 3.844 \times 10^{11}$

$2^x \geq 3.844 \times 10^{12}$

$x \geq \log_2(3.844 \times 10^{12})$

$x \geq 41.8\cdots$

The Magnitude of an Earthquake Level 2



One of the indices that can be used to express the scale of an earthquake is the magnitude M (which is unitless). Answer the following questions using the definition of M given below.

$\log_{10} E = 4.8 + 1.5M$

(Where E expresses the earthquake's energy in joules.)

(1) Find the energy E [J] of an earthquake with a magnitude of 9.

$\log_{10} E = 4.8 + 1.5M$

From the definition of a logarithm:

$E = 10^{4.8+1.5\times M}$

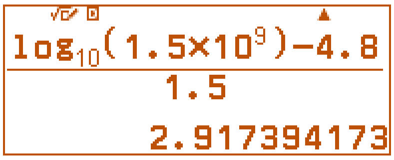
$E = 10^{4.8+1.5\times 9} \approx 2.0 \times 10^{18}$ [J]

(2) The average energy of a bolt of lightning is said to be 1.5×10^9 [J]. Find the magnitude of an earthquake that has the same amount of energy.

Since $\log_{10} E = 4.8 + 1.5M$,

$1.5M = \log_{10} E - 4.8$

$M = \dfrac{\log_{10}(1.5 \times 10^9) - 4.8}{1.5} \approx 2.9$

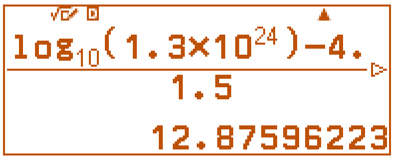


(3) The estimated impact energy of the meteorite which is thought to have caused the extinction of the dinosaurs is said to be 1.3×10^{24} (J). Find the magnitude of an earthquake with 1.3×10^{24} (J) of energy.

Since $\log_{10} E = 4.8 + 1.5M$,

$1.5M = \log_{10} E - 4.8$

$M = \dfrac{\log_{10}(1.3 \times 10^{24}) - 4.8}{1.5} \approx 12.9$



(4) How many times more energy does an earthquake have when the magnitude increases by 1?

When the magnitude, M, increases by 1, the energy, E', of an earthquake can be expressed as follows.

$E' = 10^{4.8+1.5(M+1)}$

∴The ratio of the energy when the magnitude is M compared to $M+1$ is

$\dfrac{E'}{E} = \dfrac{10^{4.8+1.5(M+1)}}{10^{4.8+1.5M}} = 10^{(4.8+1.5M+1.5)-(4.8+1.5M)} = 10^{1.5} \approx 32$

∴ 32 times

Half-life Level 3

The nuclei of the atoms of some elements sometimes emit radiation and transmute into nuclei of atoms of different elements. For these elements, the amount of time that it takes for the number of nuclei of the original element to be reduced by half is called the half-life. Letting the number of nuclei of the original element be N_0, the number of nuclei remaining after t years be N, and the half-life of the element be T years, the following relationship holds.

$$N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

(1) For the element ${}^{239}_{94}\mathrm{Pu}$ (plutonium), which has a half-life of $24{,}110$ years, what percentage of the original material remains after 100 years have elapsed?

Since $N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{100}{24{,}110}} \approx 0.997$ $\therefore 99.7\%$

(2) For the radioactive element ${}^{14}_{6}\mathrm{C}$ (carbon), which has a half-life of $5{,}730$ years, after how many years does $\frac{1}{16}$ of the original material remain?

Since $N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$,

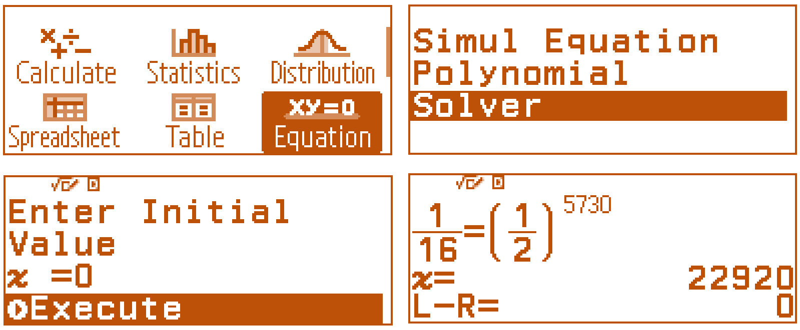
$\frac{1}{16}N_0 = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{5{,}730}}$

$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{t}{5{,}730}}$

$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{t}{5{,}730}}$

$\frac{t}{5{,}730} = 4$

$\therefore t = 4 \times 5730 = 22920$ years



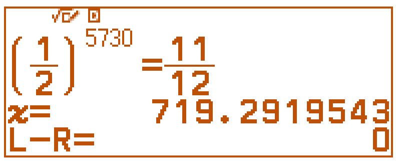
(3) Measuring the amount of ${}^{14}_{6}\mathrm{C}$ contained in a wooden artifact discovered in ancient Egyptian ruins, it was found to be $\frac{11}{12}$ that of the amount (abundance) of ${}^{14}_{6}\mathrm{C}$ in the atmosphere. Assess whether this artifact is from ancient Egyptian times. *${}^{14}_{6}\mathrm{C}$ is produced at a constant rate in the atmosphere by interaction with cosmic rays, so its abundance in the atmosphere is steady across time.

Since $N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$,

$\frac{11}{12}N_0 = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{5{,}730}}$

$\left(\frac{1}{2}\right)^{\frac{t}{5{,}730}} = \frac{11}{12}$

$\therefore t = 719$ years



$\therefore$ This artifact appears to be from a more recent time, and thus is likely not from ancient Egypt.

Relating Speed and Stopping Distance   Level 1

Assume that the relationship between the speed x [km/h] at which a certain car is traveling at the time when the brakes are applied and the stopping distance y [m] can be expressed by a quadratic function ($y = ax^2 + bx + c$)*.
The table below shows the stopping distances for the car when traveling at various speeds.

	①	②	③
Speed	20 [km/h]	30 [km/h]	40 [km/h]
Stopping Distance	9 [m]	14 [m]	21 [m]

* Assume that the relationship expressed by the quadratic function holds for speeds of 0 to 80 km/h.

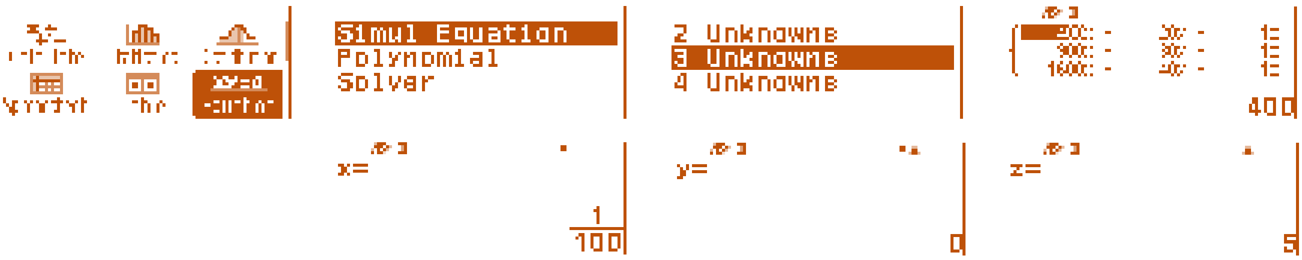
(1) Find the equation for the quadratic function that relates speed (x) and stopping distance (y).

Letting the equation we want be of the form $y = ax^2 + bx + c$,

From ①, $9 = 20^2a + 20b + c$ → $400a + 20b + c = 9$

From ②, $14 = 30^2a + 30b + c$ → $900a + 30b + c = 14$

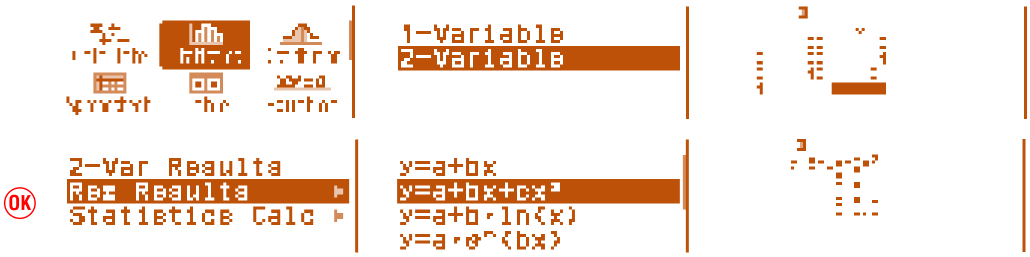
From ③, $21 = 40^2a + 40b + c$ → $1600a + 40b + c = 21$



Solving the system of simultaneous equations using a scientific calculator gives: $a = \frac{1}{100}$, $b = 0$, and $c = 5$

$\therefore y = \frac{1}{100}x^2 + 5$

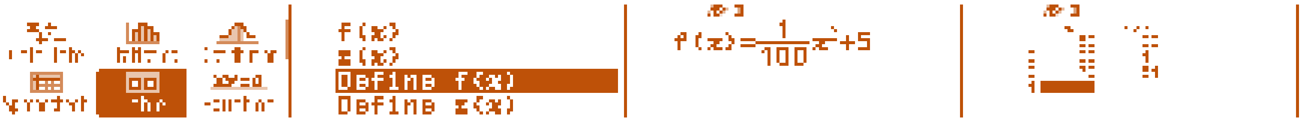
Alternate Solution: We can also find the function using the regression calculator under [Statistics].



*Note that the a and c displayed by the calculator are the reverse of those used in the calculations above.

(2) Find the stopping distance when the speed is $50\ \mathrm{km/h}$, $60\ \mathrm{km/h}$, and $70\ \mathrm{km/h}$.

Substitute $x = 50,\ 60,\ \text{and}\ 70$ into the equation we found in (1) $y = \frac{1}{100}x^2 + 5$.



Input the values directly in the x column.

Thus,

The stopping distance at $50\ \mathrm{km/h}$ is: $30\ \mathrm{m}$

The stopping distance at $60\ \mathrm{km/h}$ is: $41\ \mathrm{m}$

The stopping distance at $70\ \mathrm{km/h}$ is: $54\ \mathrm{m}$

(3) While driving at a certain speed, the driver of the car noticed an obstacle 65 m ahead and applied the brakes. What is the lowest driving speed at which the car will be unable to stop in time and hit the obstacle?

Substitute $y = 65$ into the equation we found in (1) $y = \frac{1}{100}x^2 + 5$.

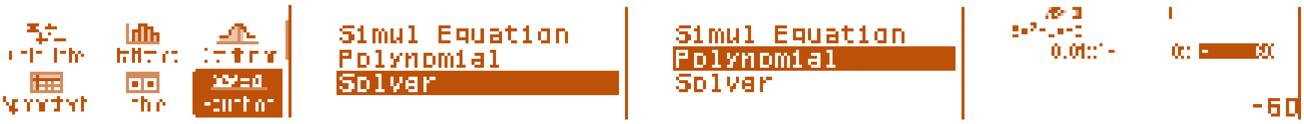
Since $65 = \frac{1}{100}x^2 + 5$,

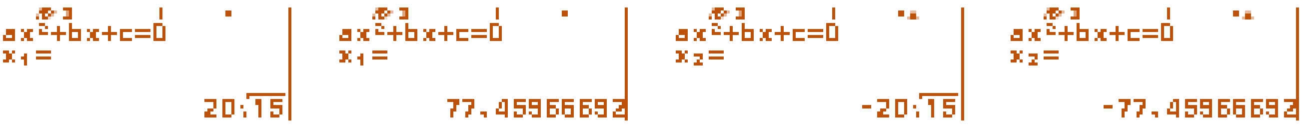
$\frac{1}{100}x^2 - 60 = 0$

$x = \pm 20\sqrt{15}$

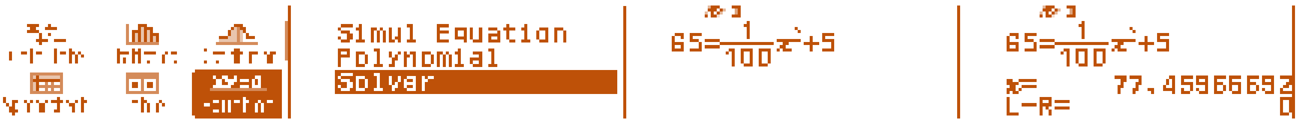
$x \approx \pm 77.46$

$\therefore\ 77.46\ \mathrm{km/h}$





Alternate Solution



Calculating an Electric Bill **Level 3**

Solve the following problems relating to the monthly electric bills for a certain apartment.

(1) To calculate the electric bill, it is necessary to calculate the energy consumption $[\mathrm{kWh}]$ for that month. Energy consumption $[\mathrm{kWh}]$ can be found by multiplying the power consumption $[\mathrm{kW}]$ value listed on an appliance by the number of hours the appliance is used $[\mathrm{h}]$.

Express the energy consumption $w(x)[\mathrm{kWh}]$ of a certain lamp with a listed power consumption of $60\ [\mathrm{W}]$ when used for x hours, in terms of x.

Since $60\ \mathrm{W} = 0.06\ \mathrm{kW}$,

Energy consumption: $w(x) =$ power consumption $[\mathrm{kW}] \times$ hours of use $[\mathrm{h}] = 0.06x\ [\mathrm{kWh}]$

(2) The monthly electric bill is calculated as the total of a fixed base rate plus a variable usage charge based on actual energy use. Given that the base rate is $\$14.00$ and the usage charge is calculated at the rate of $20¢$ ($0.20) per kWh of energy usage, express the electric bill $f(x)$ if the lamp discussed in (1) is the only electric appliance used and is run for x hours per month.

$$f(x) = \underbrace{0.2 \times w(x)}_{\text{Usage Charge}} + \underbrace{14}_{\text{Base Rate}} = 0.2 \times 0.06x + 14 = 0.012x + 14\ \ [\text{dollars}]$$

(3) Find the electric bill (in dollars) if a lamp that draws $60\ \mathrm{W}$ of power is run for 300 hours.

$$f(x) = 0.012x + 14 = 0.012 \times 300 + 14 = 17.6\ \ [\text{dollars}]$$

0.012×300+14

17.6

(4) What is the minimum number of hours that a $60\mathrm{W}$ lamp has been run if the monthly electric bill is at least \$20.00?

$0.012x + 14 \geq 20$

$\therefore x \geq 500$ [h]

（1 month $= 31$ days $\times 24$ h $= 744$ h > 500 h. Thus, this answer is reasonable.）

(5) Given that if more than $30\ \mathrm{kWh}$ of energy is used, the usage charge for energy beyond $30\ \mathrm{kWh}$ is calculated at the rate of $25¢$ (\$0.25) per kWh, express the electric bill $g(x)$ in terms of x.

$$g(x) = \underbrace{0.2 \times 30}_{\text{Usage Charge (up to 30 kWh)}} + \underbrace{0.25 \times (w(x) - 30)}_{\text{Usage Charge (over 30 kWh)}} + \underbrace{14}_{\text{Base Rate}} = 0.25 \times w(x) + 12.5 = 0.25 \times 0.06x + 12.5 = 0.015x + 12.5$$

(6) In a certain month, the resident of the apartment used more than $30\ \mathrm{kWh}$ of energy, but mistakenly calculated the payment using the equation in (2). Given that the electric company later determined that the bill was underpaid by $60¢$ (\$0.60), find the number of hours that the lamp was in use during that month.

Find the value for x such that $g(x) - f(x) = 0.6$.

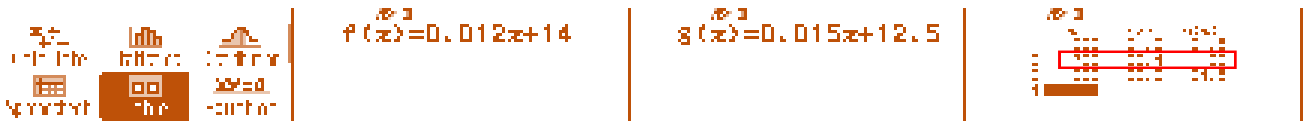
$(0.015x + 12.5) - (0.012x + 14) = 0.6$

$0.003x = 2.1$

$x = 700$ [h]

Check 1

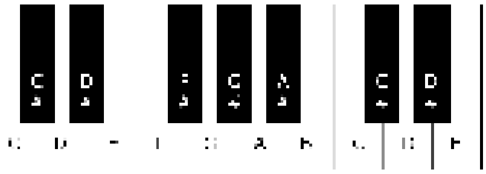
Check 2



We can see that $g(x) - f(x) = 0.6$ when x is 700 h.

Relating Musical Scales to Geometric Sequences Level 1

The range between one tone and another tone with double the frequency is called an octave, which is made up of 12 half steps. The ratio of the frequencies of any adjacent pair of tones is always constant, and the frequencies are arranged in a geometric sequence (for equally tempered tuning systems).



Scale	C	C#	D	D#	E	F	F#	G	G#
Frequency [Hz]	261.6	277.2	293.7	311.1	329.6	349.2	370.0	392.0	415.3

A	A#	B	C	C#	D	D#	E	F	F#	G	G#	A	A#
440 Standard Pitch	466.2	493.9	523.3	554.4	587.3	622.3	659.3	698.5	740.0	784.0	830.6	880	932.3

1 octave

(1) Find the common ratio for this scale.

If the first term ($n = 1$) is 440, then $n = 13$ is 880. Based on the formula for the n^{th} term in a geometric sequence $a_n = a_1 r^{n-1}$

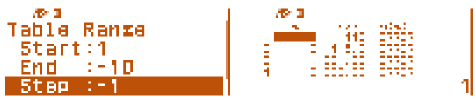
$880 = 440 \times r^{12}$

$r = \pm \sqrt[12]{2}$

It is clear from the table that $r > 0$ so, $r = \sqrt[12]{2}$

(2) The standard pitch used for tuning is an A at the frequency of 440 Hz. Find the frequencies of the other tones on the table, rounding your answer to the nearest tenth.

The steps in the Table can also be set to negative numbers.



(3) The ratios between the frequencies of the notes in a given chord are close to integers. Find the integer ratios (values very close to single-digit integers) of the frequencies in the notes C, E, and G (marked in yellow on the table).

C : E : G= $523.3 : 659.3 : 784.0 \approx 1 : 1.259 : 1.498 \approx 100 : 125 : 150 = 4 : 5 : 6$

The Tower of Hanoi

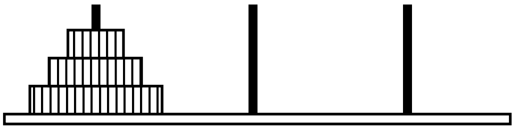
Level 3

The diagram shows a Tower of Hanoi puzzle. There are three vertical pegs, and a set of n discs, each of different sizes, is placed on the left peg in descending order of size such that the smallest disc is on top.

Let a_n be the smallest number of moves it takes to move all of the discs to the right peg while following the **Rules** given below.

Rules

- Only one disc may be moved at a time.
- Discs can only be moved from the top of one stack onto another peg.
- No larger disc can be placed on top of a smaller disc.



(1) Find a_n for $n = 1$ through $n = 4$.

Number of Discs	$n = 1$	$n = 2$	$n = 3$	$n = 4$
Number of Moves a_n	1	3	7	15

*It is easy to experiment with this concept using cups or other items of differing sizes.

(2) Using the pattern you found in (1), guess the relationship between the number of moves required when there are n discs, a_n, and the number when there are $n-1$ discs, a_{n-1}. (Find the recurrence formula.)

From (1), we can see that the value of a_n is double a_{n-1} plus 1.

$\therefore a_n = 2a_{n-1} + 1$

(Proof)

The following operations are required to move a Tower of Hanoi with n discs:

1. Move all of the discs other than the largest disc ($n-1$ discs) to the center peg (requiring a_{n-1} moves).
2. Move the largest disc to the right peg (requiring 1 move)
3. Move the $n-1$ discs from the center peg to the right peg, on top of the largest disc (requiring a_{n-1} moves).

$\therefore a_n = a_{n-1} + 1 + a_{n-1} = 2a_{n-1} + 1$

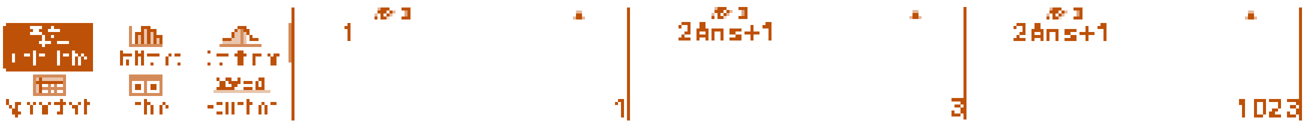
(3) Find a_n when $n = 5$ and $n = 10$.

$a_5 = 2a_4 + 1 = 2 \times 15 + 1 = 31$ (moves)

This problem is simple to solve using your scientific calculator's Answer function.

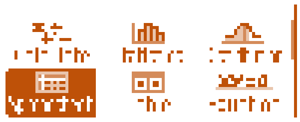
$a_{10} = 1023$ (moves)

1 EXE 2 Ans + 1 EXE EXE EXE EXE EXE EXE EXE EXE EXE



(4) Find a_n when $n = 30$.

This problem is simple to solve using a spreadsheet on your scientific calculator.



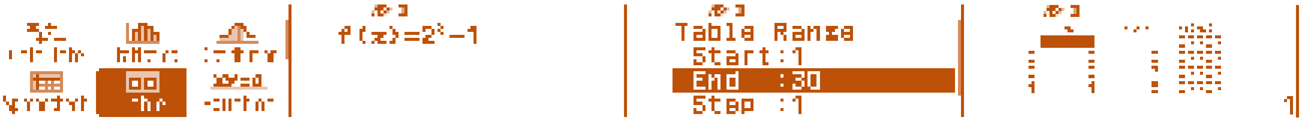
When $n = 30$, $a_{30} = 1073741823$ (moves)

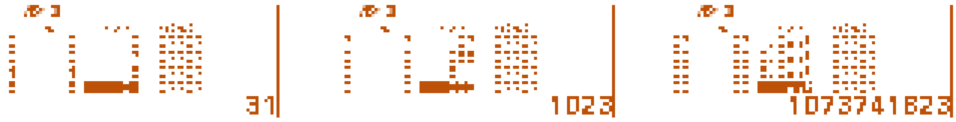
(5) Find the formula for the n^{th} term for moves required, a_n, when there are n discs. (Express a_n in terms of n.)

Focus on the fact that each term is equal to 2 to the n^{th} power minus 1.

$a_n = 2^n - 1$

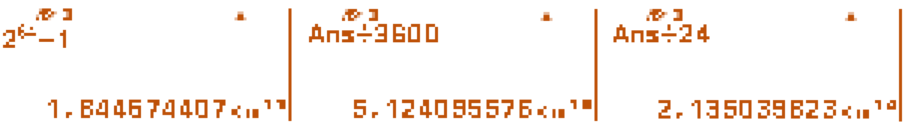
Check using your scientific calculator's Table function.



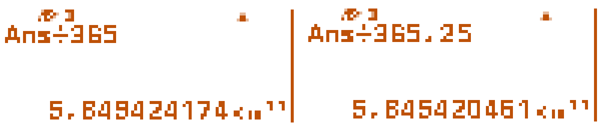


Check (3) Check (3) Check (4)

(6) French mathematician Édouard Lucas purportedly recounted a legend stating that the world would end when a certain Tower of Hanoi with 64 discs had been completed. Assuming that the puzzle is solved according to the rules, and that one move is made every second, approximately how many years would it take to complete a Tower of Hanoi with 64 discs ($n = 64$)?



Seconds Hours Days



Years (Ignoring leap years)

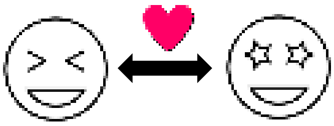
Years (Accounting for leap years)

∴ It would take about 584.5 billion years.

Matching Based on Similarity

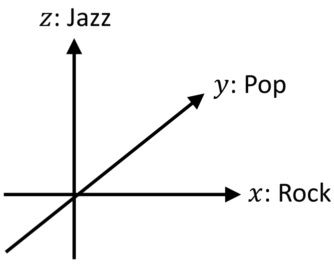
Level 2

To find classmates with similar music preferences, the top 100 songs in the playlists of students A, B, C, and D were categorized into three genres: rock, pop, and jazz. The results are as follows. Identify the pair with the most similar preferences and the pair with the most different preferences. Explain using vector analysis.

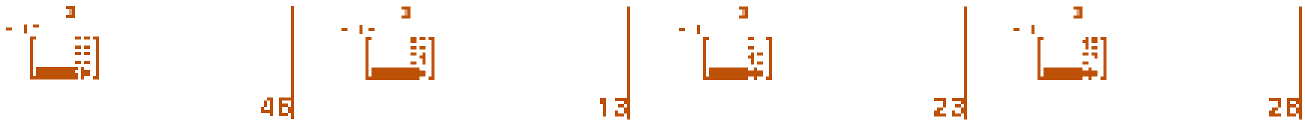


Students	Rock	Pop	Jazz
A	26	28	46
B	53	34	13
C	31	46	23
D	45	27	28

Hint: By representing each value in the vector space shown on the right, you can evaluate their preferences.



As shown on the right, plot rock on the x-axis, pop on the y-axis, and jazz on the z-axis. Smaller angles between the 3D vectors indicate more similar preferences.



Pair	AB (The most different preferences)	AC	AD
Angle			
Pair	BC	BD (The most similar preferences)	CD
Angle			

Finding the Shortest Flight Path for a Drone   Level 2

Four points, A $(3,\ 6,\ 7)$, B $(-8,\ 5,\ -4)$, C $(-7,\ -5,\ 3)$, and D $(5,\ 3,-6)$, lie in the same three-dimensional space. A drone will be launched from A and make deliveries at B, C, and D, but the deliveries can be made in any order. From among the possible flight paths including B, C, and D and returning to A, let's consider the flight path with the shortest total travel distance.

(1) Assuming the flight path consists only of straight lines connecting the various points, give all possible flight paths.

E.g.) Write ABCDA if the path is A → B → C → D → A.

ABCDA   ABDCA   ACBDA   ACDBA   ADBCA   ADCBA

(Check)

The total number of permutations for ordering B, C, and D is 3!

(2) Which of the flight paths from (1) has the shortest total travel distance?

Input the following values into your scientific calculator.

Vct A $= (3,\ 6,\ 7)$, Vct B $= (-8,\ 5,\ -4)$, Vct C $= (-7,\ -5,\ 3)$, Vct D $= (5, 3, -6)$

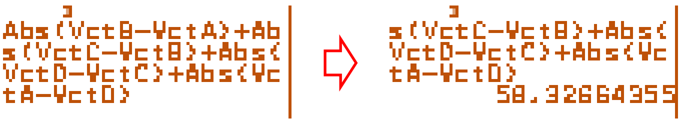
Since reversed paths have the same total travel distance,

ABCDA = ADCBA, ABDCA = ACDBA, and ACBDA = ADBCA, so

$\mathrm{ABCDA} = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CD}| + |\overrightarrow{DA}|$

$= |\vec{B} - \vec{A}| + |\vec{C} - \vec{B}| + |\vec{D} - \vec{C}| + |\vec{A} - \vec{D}|$

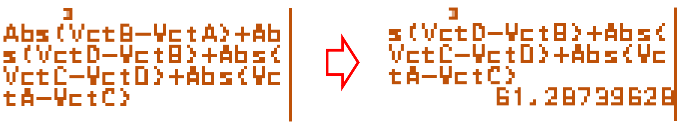
≈ 58.3



$\mathrm{ABDCA} = |\overrightarrow{AB}| + |\overrightarrow{BD}| + |\overrightarrow{DC}| + |\overrightarrow{CA}|$

$= |\vec{B} - \vec{A}| + |\vec{D} - \vec{B}| + |\vec{C} - \vec{D}| + |\vec{A} - \vec{C}|$

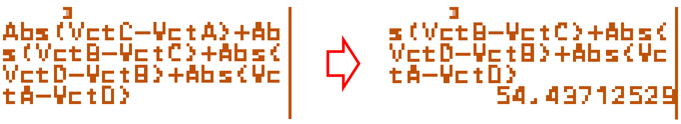
≈ 61.3



$\mathrm{ACBDA} = |\overrightarrow{AC}| + |\overrightarrow{CB}| + |\overrightarrow{BD}| + |\overrightarrow{DA}|$

$= |\vec{C} - \vec{A}| + |\vec{B} - \vec{C}| + |\vec{D} - \vec{B}| + |\vec{A} - \vec{D}|$

≈ 54.4



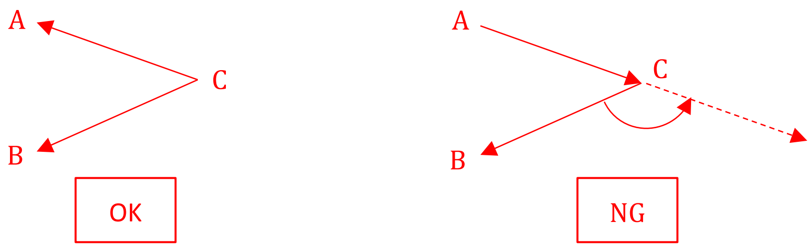
Thus, the path with the shortest total travel distance is ACBDA or ADBCA.

(3) For the shortest flight path found in (2), find the angle for each of the turns along the path.

First, let the angle of the turn at C in path $A \to C \to B \to D \to A$ be $\angle ACB$.

To find the measure of $\angle ACB$, use your scientific calculator to find the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$, which have the same initial point, as shown in the diagram.

(*Be careful not to find the angle between $\overrightarrow{AC}$ and $\overrightarrow{CB}$ by following the route, as shown.)



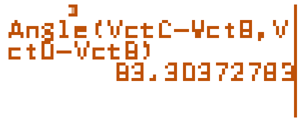
- $\angle ACB$: angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$ = angle between $(\vec{A}-\vec{C})$ and $(\vec{B}-\vec{C})$

Calculating with your scientific calculator gives $\angle ACB = 67.6°$.

Calculating the measures of the other angles in the same manner gives:

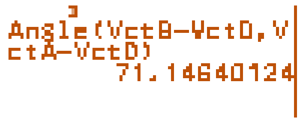
- $\angle CBD$: angle between $\overrightarrow{BC}$ and $\overrightarrow{BD}$ = angle between $(\vec{C}-\vec{B})$ and $(\vec{D}-\vec{B})$

$\therefore \angle CBD = 83.3°$



- $\angle BDA$: angle between $\overrightarrow{DB}$ and $\overrightarrow{DA}$ = angle between $(\vec{B}-\vec{D})$ and $(\vec{A}-\vec{D})$

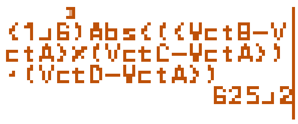
$\therefore \angle BDA = 71.1°$

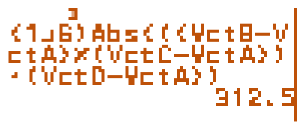


(4) Find the volume V of the air space through which the drone flies (the volume of tetrahedron ABCD).

Formula for the volume of a tetrahedron:

$$V = \frac{1}{6} \times \left|(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}\right| = \frac{1}{6} \times \left|\left((\vec{B}-\vec{A}) \times (\vec{C}-\vec{A})\right) \cdot (\vec{D}-\vec{A})\right| = 312.5$$





Total score of university entrance exams Level 1

(1) Table 1 shows the raw scores of high school students X, Y, and Z in the university entrance exams for three subjects: mathematics, English, and science. Find the total raw scores of X, Y, and Z for each of the three subjects.

Table 1. Raw score table

	Mathematics	English	Science
X	80	80	80
Y	90	60	90
Z	90	70	80

Total raw score of X $= 80 + 80 + 80 = 240$

Total raw score of Y$= 90 + 60 + 90 = 240$

Total raw score of Z$= 90 + 70 + 80 = 240$

(2) Table 2 shows the weighting of subjects used to evaluate high school students' grades at three universities, P, Q, and R. Each university multiplies the raw score of each student's subject by the corresponding value in Table 2, and the sum of the three subjects is the final total score. At this point, use matrix calculations to find each student's total score and complete Table 3. Also, which of the three students has the highest final total score at each university?

Table 2. Subject weighting

	P Uni.	Q Uni.	R Uni.
Mathematics	10	9	8
English	9	8	10
Science	8	10	9

Table 3. Final Total Score

	P Uni.	Q Uni.	R Uni.
X	2160	2160	2160
Y	2160	2190	2130
Z	2170	2170	2140

By multiplying the matrices in Table 1 and Table 2, we can find each student's final total score at each university.

$$\begin{bmatrix} 80 & 80 & 80 \\ 90 & 60 & 90 \\ 90 & 70 & 80 \end{bmatrix} \begin{bmatrix} 10 & 9 & 8 \\ 9 & 8 & 10 \\ 8 & 10 & 9 \end{bmatrix} = \begin{bmatrix} 2160 & 2160 & 2160 \\ 2160 & 2190 & 2130 \\ 2170 & 2170 & 2140 \end{bmatrix}$$

Therefore, the students with the highest final total scores at each university are as follows.

University P: Z, University Q: Y, University R: X

Cryptography using matrices **Level 2**

There is a method of transmitting information by encrypting (matrix calculation) numbers assigned to letters, as shown below.

Letter	A	B	C	D	E	F	G	H	I	J
Number	11	12	13	14	15	16	17	18	19	20

K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

Letter $\begin{bmatrix} M & A \\ T & H \end{bmatrix}$ → Number Allocation → *Number* (matrix) $\begin{bmatrix} 23 & 11 \\ 30 & 18 \end{bmatrix}$ → Coding → *Code* (matrix) $\begin{bmatrix} 311 & 167 \\ 182 & 98 \end{bmatrix}$

Use the table above

Multiply a *Key* (matrix) by the front of the *Number* to obtain the *Code*.

$Key \times Number = Code$ ……①

$$\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 23 & 11 \\ 30 & 18 \end{bmatrix} = \begin{bmatrix} 311 & 167 \\ 182 & 98 \end{bmatrix}$$

※ In this example, the *Key* is $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$

(1) Decode the *Code* $\begin{bmatrix} 236 & 235 \\ 138 & 138 \end{bmatrix}$ by using the *Key* $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$.

① can be transformed as follows:

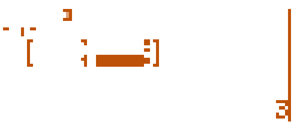
$[Key][Number] = [Code]$ ……①

$[Key]^{-1}[Key][Number]=[Key]^{-1}[Code]$

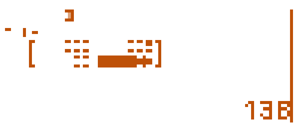
$[Number]=[Key]^{-1}[Code]$

Therefore, to find the *Number*, we can multiply the inverse of the *Key* from the front of the *Code*.

$$[Number] = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 236 & 235 \\ 138 & 138 \end{bmatrix} = \begin{bmatrix} 18 & 15 \\ 22 & 26 \end{bmatrix}$$

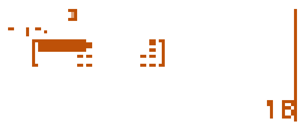


Input the *Key* to Mat *A*



Input the *Code* to Mat *B*

Enter the formula



Number

Assigning *Letters* from the table $\begin{bmatrix} 18 & 15 \\ 22 & 26 \end{bmatrix} = \begin{bmatrix} H & E \\ L & P \end{bmatrix}$ → HELP

(2) Information was leaked that when the $Code$ $\begin{bmatrix} 328 & 133 \\ 209 & 85 \end{bmatrix}$ is decoded, the letters are "SAVE". Find the Key (matrix) in this case.

① can be transformed as follows.

$[Key][Number] = [Code]$ ……①

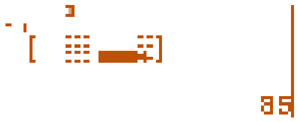
$[Key][Number]\ [Number]^{-1} = [Code][Number]^{-1}$

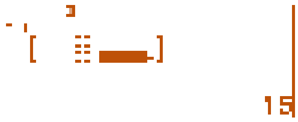
$[Key] = [Code][Number]^{-1}$

Therefore, to obtain the Key, we can multiply the inverse of the $Number$ from the back of the $Code$.

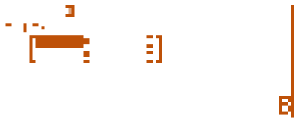
Since the $Number$ of SAVE is $\begin{bmatrix} 29 & 11 \\ 32 & 15 \end{bmatrix}$ from the table,

$$[Key] = \begin{bmatrix} 328 & 133 \\ 209 & 85 \end{bmatrix} \begin{bmatrix} 29 & 11 \\ 32 & 15 \end{bmatrix}^{-1} = \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$$


Input $Code$ to Mat C


Input $Number$ to Mat D


Enter the formula

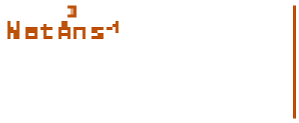

Key

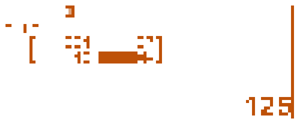
(3) Decode the $Code$ $\begin{bmatrix} 224 & 197 \\ 142 & 125 \end{bmatrix}$ using the Key obtained in (2).

As in (1), to obtain the $Number$, we can multiply the inverse of the Key from the front of the $Code$.

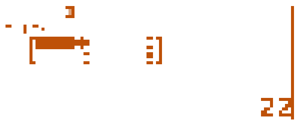
$$[Number] = \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 224 & 197 \\ 142 & 125 \end{bmatrix} = \begin{bmatrix} 22 & 19 \\ 16 & 15 \end{bmatrix}$$


Enter Mat Ans for Key in (2)


Inverse matrix


Enter $Code$ in Mat A


Enter the formula


$Number$

Assigning $Letters$ from the table, $\begin{bmatrix} \mathrm{L} & \mathrm{I} \\ \mathrm{F} & \mathrm{E} \end{bmatrix}$ → LIFE

(4) Decode the $Code$ $\begin{bmatrix} 66 & 82 & 64 & 30 \\ 14 & 45 & 16 & 17 \\ 11 & 21 & 34 & 28 \\ 53 & 175 & 96 & 96 \end{bmatrix}$ by using the Key $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & 5 \end{bmatrix}$.

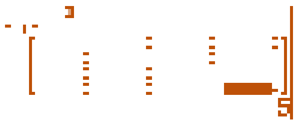
However, the following table should be used for this question.

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Number	0	1	2	3	4	5	6	7	8	9	10	11	12

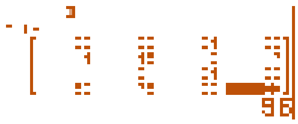
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
13	14	15	16	17	18	19	20	21	22	23	24	25	26

As in (1), to obtain the $Number$, we can multiply the inverse of the Key from the front of the $Code$.

$$Number = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 66 & 82 & 64 & 30 \\ 14 & 45 & 16 & 17 \\ 11 & 21 & 34 & 28 \\ 53 & 175 & 96 & 96 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 18 & 8 \\ 14 & 26 & 2 & 0 \\ 11 & 2 & 20 & 11 \\ 0 & 19 & 14 & 17 \end{bmatrix}$$

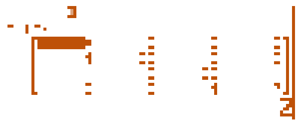


Input Key to Mat A



Enter $Code$ in Mat B

Enter the formula



$Number$

Assigning $Letters$ from the table, $\begin{bmatrix} C & A & S & I \\ O & \_ & C & A \\ L & C & U & L \\ A & T & O & R \end{bmatrix}$ → CASIO_CALCULATOR

Comparing Histograms

Level 1

Each of the students in two physical education classes, Class A and Class B measured their grip strength. The results for each of the students are given below.

【Class A: 34 Students】

39, 36, 37, 38, 38, 41, 38, 37, 40, 39, 43, 38, 38, 40, 39, 40, 39, 42, 41, 36, 39, 42, 38, 37, 43, 37, 37, 39, 38, 43, 36, 36, 39, 38 (kg)

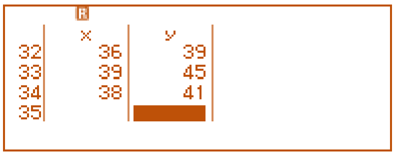
【Class B: 34 Students】

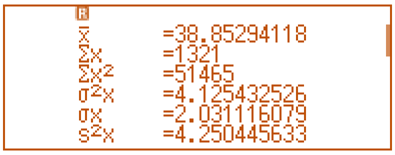
36, 30, 31, 40, 33, 34, 35, 45, 35, 46, 26, 36, 48, 39, 39, 36, 30, 45, 41, 42, 42, 43, 44, 35, 46, 37, 48, 49, 48, 36, 46, 39, 45, 41 (kg)

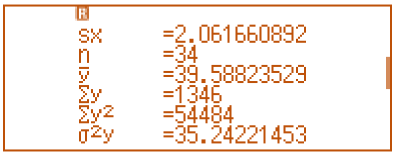
Answer the following questions using this data.

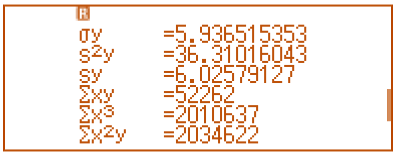
(1) Find the mean and standard deviation for Class A and Class B. Then, explain the characteristics of the means and standard deviations of the two classes.

Input the results for Class A into column x and the results for Class B into column y.







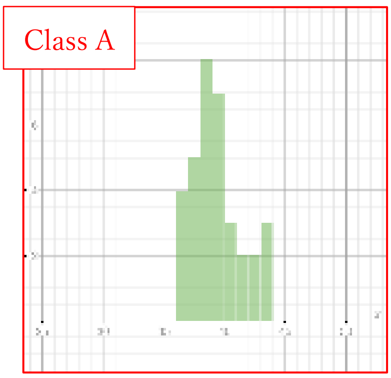


Class A: The mean is 38.85 kg. / The standard deviation is 2.03 kg.

Class B: The mean is 39.59 kg. / The standard deviation is 5.94 kg.

(Characteristics) The means are nearly the same, but Class B has a larger standard deviation, which implies that there is more variability in Class B's data.

(2) Create histograms for Class A and Class B, setting the bin width at 1 kg.



(3) Analyze the histograms created in (2) and explain the data trends in the observed results for each class. Then, consider the reasons why these data trends may have appeared.

The majority of the results are clustered close to the mean of 38.85 kg, which shows that the results for most of the students are close to the mean.

Thus, for Class A, the mean and the mode are near each other, and the variability in the data is small.

These data trends may be explained by the fact that there is not much variation in physical strength among the students in Class A, most of whom have average grip strength.

The data is scattered, with many of the values far from the mean of 39.59 kg and some bins having zero students.

Thus, one look at the histogram shows that there is large variability in the data for Class B.

These data trends may be explained by the fact that there is more variation in physical strength among the students in Class B compared to those in Class A.

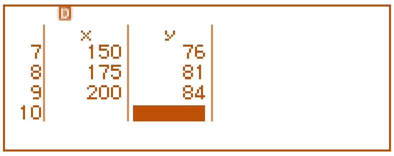
Linear Regression Analysis **Level 1**

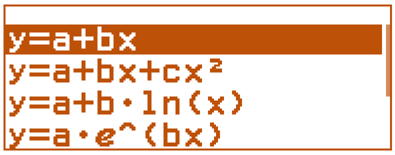
A student attempted to verify whether there is a relationship between caffeine intake and attention span.

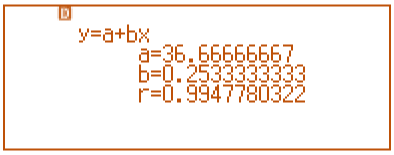
The table below shows the results of the student's observations.

Caffeine Intake (mg)	Attention Span (minutes)
0	35
25	42
50	49
75	57
100	64
125	70
150	76
175	81
200	84

(1) Letting x be caffeine intake and y be attention span, find the linear regression equation, $y = bx + a$, and correlation coefficient, r. Then, draw the regression line.



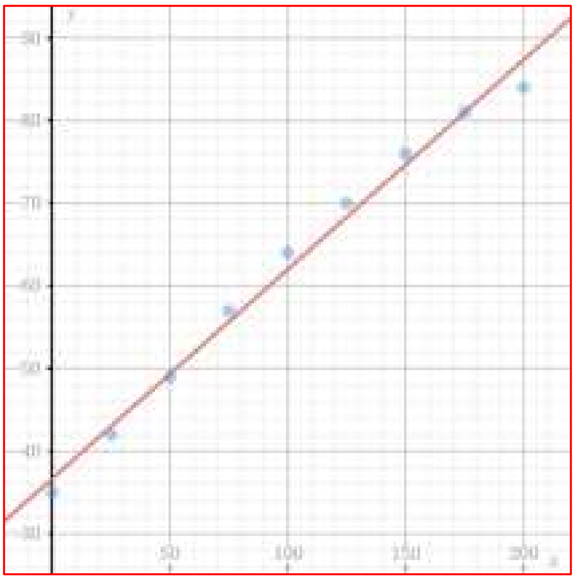




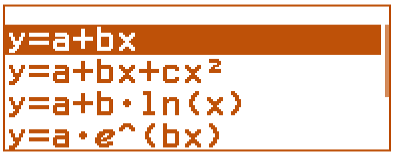
The linear regression equation is $y = 0.25x + 36.67$.

The correlation coefficient, r, is 0.99.

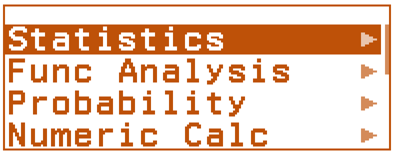
Return to the screen where you entered the values and press ⇧ then ⓧ to display the QR code.

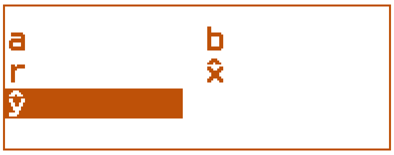


(2) Project the attention span following 160 mg of caffeine intake using the regression line found in (1). Then, find the difference between the actual attention span as measured and the projected attention span for 100 mg of caffeine intake.



Press and select the function that you want to use.





The projected attention span following 160 mg of caffeine intake is 77.2 minutes.

The difference between the actual attention spans as measured and the projected attention span for 100 mg of caffeine intake is extremely short at only 2 minutes.

(3) Determine whether caffeine intake and attention span are correlated.

In (1), we found that the correlation coefficient is 0.99, which suggests an extremely strong positive correlation, and thus, caffeine intake and attention span are correlated.

(Note) From (1), we can see that all of the points representing measured values lie close to the regression line, which indicates that the differences between measured and predicted values are small, and that the data shows a strong correlation. However, despite this strong correlation, it is important to avoid overconsumption of caffeine, which is harmful. (For reference, the recommended maximum daily intake for a non-pregnant adult is 400 mg.)

Number of Correct Answers When Randomly Selecting Test Responses　Level 2

Consider a set of 10 problems, each with only 1 correct answer among 4 possible answer choices.

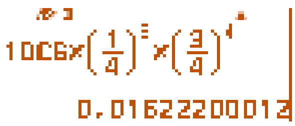
(1) Let X be the number of correct answers and $P(X)$ be the probability of X when each response is randomly selected. Complete the table.

Number of Correct Answer X	0	1	2	3	4	5	6	7	8	9	10
Probability $P(X)$	0.0563	0.1877	0.2815	0.2502	0.1459	0.0583	0.0162	3×10^{-3}	3.8×10^{-4}	2.8×10^{-5}	9.5×10^{-7}

・Method 1

Example) The probability of getting 6 correct answers is

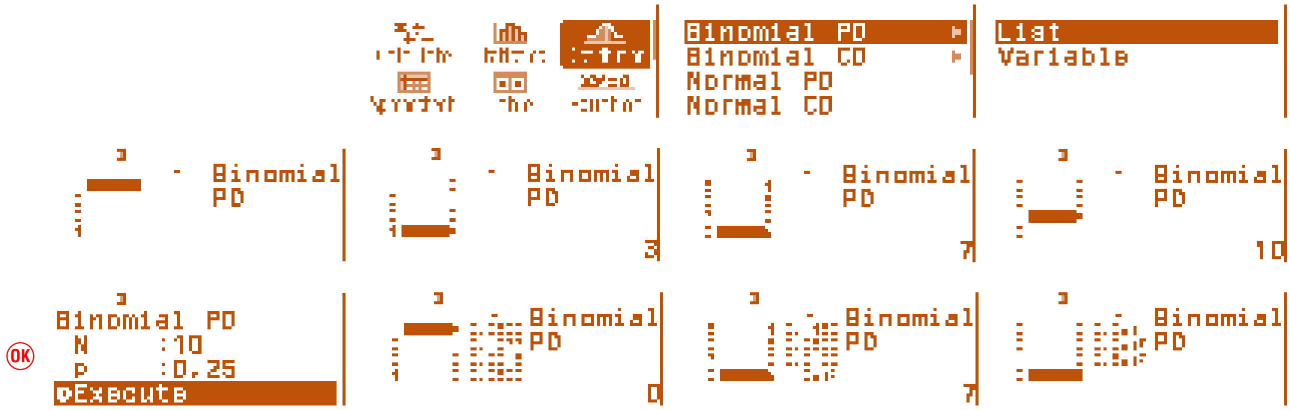
$P(6) = {}_{10}C_6 \times (\frac{1}{4})^6 \times (\frac{3}{4})^4 \approx 0.01622$ (Approximately 1.6%)



Use the same method to calculate the other probabilities $P(X) = {}_{10}C_X \times (\frac{1}{4})^X \times (\frac{3}{4})^{10-X}$.

・Method 2

You can use the [Distribution] function to create a binomial distribution (probability distribution) table.



・Method 3

[Table], ⒡ [Define f(x)], input $f(x) = {}_{10}C_x \times (\frac{1}{4})^x \times (\frac{3}{4})^{10-x}$, input the value of x (0~10) in the table.

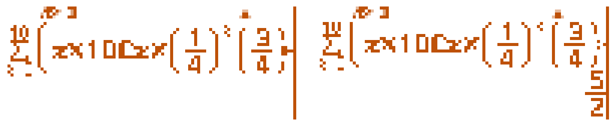
(2) Find the expected value for the number of correct answers when randomly selecting responses.

・Method 1

From the definition of an expected value:

$$0\cdot P(0)+1\cdot P(1)+2\cdot P(2)+3\cdot P(3)+4\cdot P(4)+5\cdot P(5)+6\cdot P(6)+7\cdot P(7)+8\cdot P(8)+9\cdot P(9)+10\cdot P(10)$$

$$=\sum_{X=0}^{10} X\cdot P(X)=\sum_{X=0}^{10} X\cdot {}_{10}C_X\times\left(\frac{1}{4}\right)^X\times\left(\frac{3}{4}\right)^{10-X}=2.5$$



・Method 2

The expected value of a binomial distribution (number of trials: n, probability: p) is np.

Thus, $np=10\times\frac{1}{4}=2.5$.

(3) Find the standard deviation of the number of correct answers when randomly selecting responses.

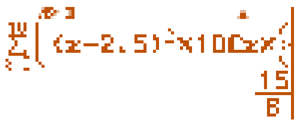
・Method 1

From the definition of a variance:

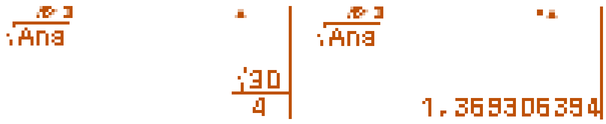
Variance$=(0-2.5)^2\cdot \mathrm{P}(0)+(1-2.5)^2\cdot \mathrm{P}(1)+(2-2.5)^2\cdot \mathrm{P}(2)+(3-2.5)^2\cdot \mathrm{P}(3)+(4-2.5)^2\cdot \mathrm{P}(4)$

$+(5-2.5)^2\cdot \mathrm{P}(5)+(6-2.5)^2\cdot \mathrm{P}(6)+(7-2.5)^2\cdot \mathrm{P}(7)+(8-2.5)^2\cdot \mathrm{P}(8)+(9-2.5)^2\cdot \mathrm{P}(9)+(10-2.5)^2\cdot \mathrm{P}(10)$

$$=\sum_{X=0}^{10}(X-2.5)^2\cdot P(X)=\sum_{X=0}^{10}(X-2.5)^2\cdot {}_{10}C_X\times\left(\frac{1}{4}\right)^X\times\left(\frac{3}{4}\right)^{10-X}=\frac{15}{8}$$



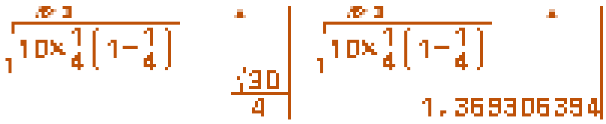
$\therefore$ Standard deviation$=\sqrt{\text{Variance}}=\sqrt{\frac{15}{8}}=\frac{\sqrt{30}}{4}\approx 1.37$



・Method 2

The standard deviation of a binomial distribution (number of trials: n, probability: p) is $\sqrt{np(1-p)}$.

Thus, $\sqrt{np(1-p)}=\sqrt{10\times\frac{1}{4}(1-\frac{1}{4})}\approx 1.37$

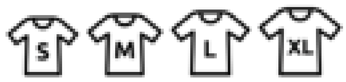


Estimating T-Shirt Production Levels Using Height Distribution　Level 2

A certain clothing manufacturer is planning to produce a total of 1,000 men's T-shirts. If the number of shirts of each size that should be produced is estimated using the probability distribution for height among men in the country, find the approximate number of shirts of each size that should be produced using the information below.

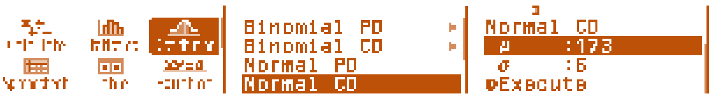
・This table shows the recommended height range for each shirt size.

Size	S	M	L	XL
Height [cm]	Up to 165	165 to 175	175 to 185	185 and up



・Heights among men in the country follow a standard normal distribution, with a mean (μ) of 173 cm, and standard deviation (σ) of 6.0 cm.

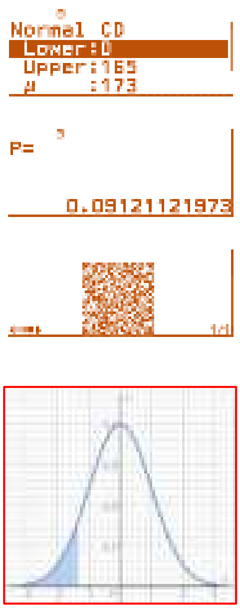
Find each of the probabilities using the [Distribution (Normal CD)] function on your scientific calculator.



μ (mean): input 173
σ (standard deviation): input 6

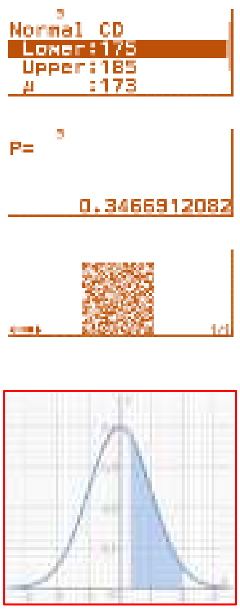
*The actual upper and lower bounds of the standard normal distribution are ∞ and −∞, respectively.

S: Lower: input 0*
Upper: input 165

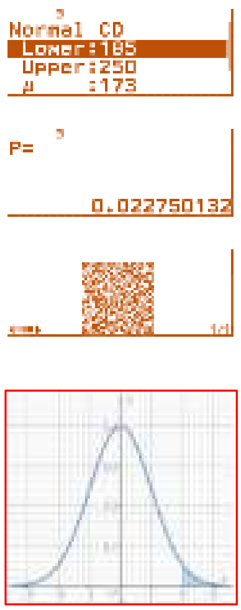


M: Lower: input 165
Upper: input 175

L: Lower: input 175
Upper: input 185



XL: Lower: input 185
Upper: input 250*



Size	S	M	L	XL
Height [cm]	Up to 165	165 to 175	175 to 185	185 to 195
Probability	0.09121	0.53935	0.34669	0.02275
Quantity 1000 ×Probability	91.21 ≈ 91	539.35 ≈ 539	346.69 ≈ 347	22.75 ≈ 23

Aging and the Apparent Passage of Time Level 1

Some people feel that time seems to pass more quickly as they age. Referencing this feeling, it has been said that "the apparent length of an interval at a given point in a person's life is proportional to the reciprocal of the person's age," (Janet's Law). Based on this concept, the apparent length of a particular interval (y) for a person at a certain age (x) can be calculated as follows.

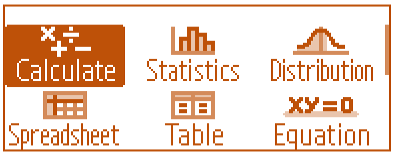
$$y = \frac{1}{x+1}$$

(1) When comparing the apparent length of a given interval of time for a person who is 10 years old and the apparent length of the same interval for a person who is 50 years old, how many times shorter is the 50-year-old's perception of the interval?

The apparent length of an interval of time for a 10-year-old is $\frac{1}{10+1} = \frac{1}{11}$

The apparent length of an interval of time for a 50-year-old is $\frac{1}{50+1} = \frac{1}{51}$

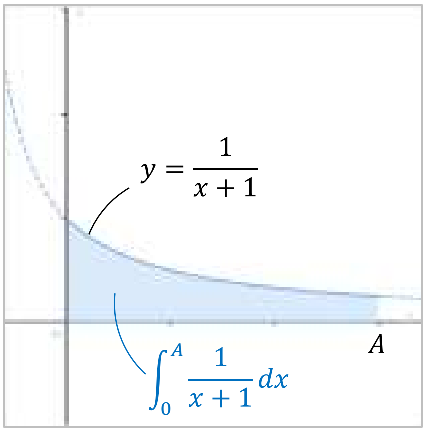
Thus, $\frac{1}{11} \div \frac{1}{51} = 4.63636\ldots$



Therefore, a 50-year-old perceives time as passing 4.6 times faster than a 10-year-old.

(2) Applying this formula for the apparent length of a given interval of time and assuming an estimated life expectancy of 85 years, what percentage of your apparent lifetime have you already experienced? Answer using your own current age.

Note that the apparent length of the interval of time from age 0 to age A is calculated as $\int_0^A \frac{1}{x+1} dx$ (the area under the curve in the diagram to the right).

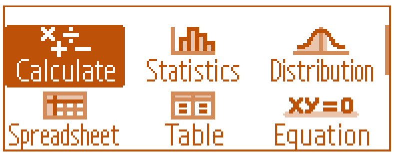


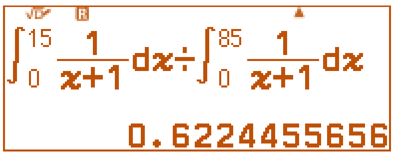
(If you are 15 years old)

The apparent length of the interval from age 0 to age 15 is $\int_0^{15} \frac{1}{x+1} dx$

The apparent length of the interval from age 0 to age 85 is $\int_0^{85} \frac{1}{x+1} dx$

Thus, $\int_0^{15} \frac{1}{x+1} dx \div \int_0^{85} \frac{1}{x+1} dx = 0.62244\ldots$





Therefore, if you are 15 years old now, you have experienced 62% of the apparent length of your life.

*Changing 15 to your actual age will give the apparent rate of progression for your life.

Cutting a Heart-Shaped Pizza in Half **Level 2**

You want to cut a heart-shaped pizza into two equal pieces, but you don't want to cut it vertically, as shown by the blue line in Figure 1. You come up with the idea of cutting the pizza in half horizontally, as shown by the green line in Figure 2. Assume that the shape of the pizza is defined by the graphs shown in Figure 3 and Figure 4.

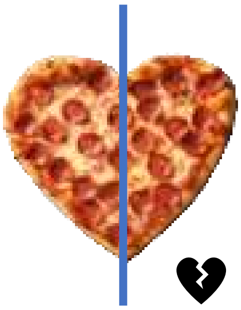


Fig. 1

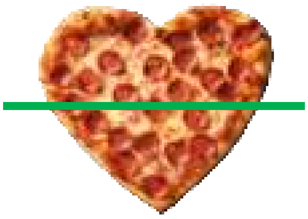


Fig. 2

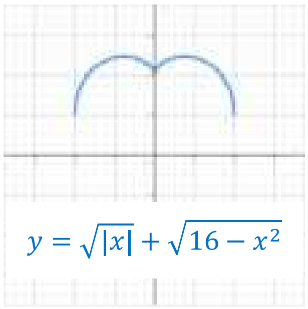


Fig. 3

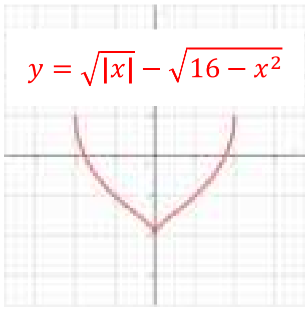
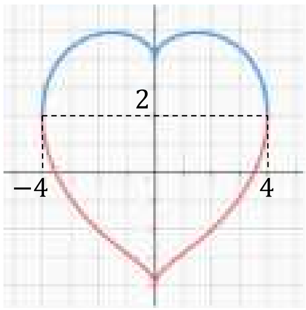
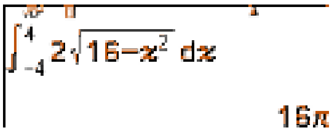


Fig. 4

(1) Find the area of the heart-shaped pizza as shown in the diagram to the right.

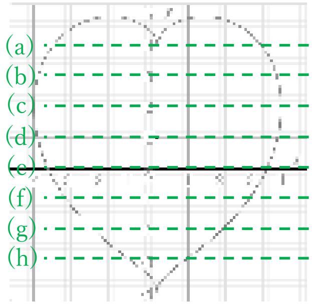


$\int_{-4}^{4}\left\{\left(\sqrt{|x|}+\sqrt{16-x^2}\right)-\left(\sqrt{|x|}-\sqrt{16-x^2}\right)\right\}dx = \int_{-4}^{4} 2\sqrt{16-x^2}\,dx = 16\pi$



*This computation may take some time for your calculator to complete.

(2) When cutting the pizza in half horizontally as shown in Figure 2, which of the lines to the right (a) – (h) would be the most appropriate? Explain your reasoning.



First, we know from (1) that the area of the pizza is 16π, so cutting the pizza in half gives:

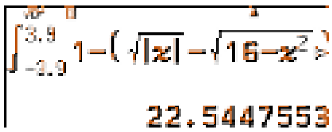
$16\pi \div 2 = 8\pi = 25.13274...$ (A)

Next, consider cutting the pizza at line (d).

From the graph, we can see that the x-coordinates of the intersections between line (d) ($y = 1$) and $y = \sqrt{|x|} - \sqrt{16 - x^2}$ are approximately 3.9 and −3.9.

Accordingly, calculating the approximate area of the pizza below line (d) gives:

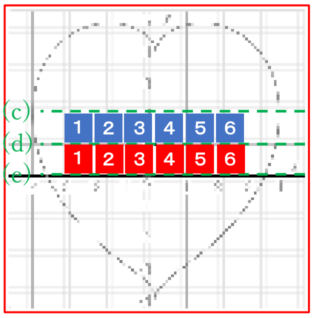
$\int_{-3.9}^{3.9}\left\{1-\left(\sqrt{|x|}-\sqrt{16-x^2}\right)\right\}dx = 22.54475...$ (B)



*The computation for (B) may take some time for your calculator to complete.

From the diagram to the right, we can see that the approximate area of the pizza below line (c) is over 6 square units greater than the area of (B), meaning that it will be over 28.5 .... (C)

Similarly, from the diagram to the right, we can see that the approximate area of the pizza below line (e) is over 6 square units less than the area of (B), meaning that it will be under 16.5 .... (D)



Thus, from (A) – (D), we can see that cutting the pizza at line (d) will give the most even pieces.

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